Intelligence Integration in Distributed Knowledge Management

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Chapter IV
Application of Uncertain Variables to Knowledge-Based Resource Distribution

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ABSTRACT

The chapter concerns a class of systems composed of operations performed with the use of resources allocated to them. In such operation systems, each operation is characterized by its execution time depending on the amount of a resource allocated to the operation. The decision problem consists in distributing a limited amount of a resource among operations in an optimal way, that is, in finding an optimal resource allocation. Classical mathematical models of operation systems are widely used in computer supported projects or production management, allowing optimal decision making in deterministic, well-investigated environments. In the knowledge-based approach considered in this chapter, the execution time of each operation is described in a nondeterministic way, by an inequality containing an unknown parameter, and all the unknown parameters are assumed to be values of uncertain variables characterized by experts. Mathematical models comprising such two-level uncertainty are useful in designing knowledge-based decision support systems for uncertain environments. The purpose of this chapter is to present a review of problems and algorithms developed in recent years, and to show new results, possible extensions and challenges, thus providing a description of a state-of-the-art in the field of resource distribution based on the uncertain variables.

INTRODUCTION

Among many theories of uncertainty (Klir, 2006) developed for different applications the uncertain variables introduced by Bubnicki (2001a, 2001b) may be considered as a useful tool for modeling expert's knowledge in knowledge-based decision systems. In the definition of the uncertain variable $\bar{x}$ we consider two soft properties: "$\bar{x} \cong x$" which means "$\bar{x}$ is approximately equal to x" or "x is the approximate value of $\bar{x}$," and "$\bar{x} \,\tilde{\in}\, D_x$" which means "$\bar{x}$ approximately belongs to the set D_x" or "the approximate value of $\bar{x}$ belongs to D_x." The *uncertain variable* $\bar{x}$ is defined by a set of values X (real number vector space), the function $h(x) = v(\bar{x} \cong x)$ (i.e., the *certainty index* that $\bar{x} \cong x$, given by an expert) and the following definitions for $D_x, D_1, D_2 \subseteq X$:

$$v(\bar{x} \,\tilde{\in}\, D_x) = \max_{x \in D_x} h(x)$$

$$v(\bar{x} \,\tilde{\notin}\, D_x) = 1 - v(\bar{x} \,\tilde{\in}\, D_x),$$

$$v(\bar{x} \,\tilde{\in}\, D_1 \vee \bar{x} \,\tilde{\in}\, D_2) = \max\{v(\bar{x} \,\tilde{\in}\, D_1), v(\bar{x} \,\tilde{\in}\, D_2)\},$$

$$v(\bar{x} \,\tilde{\in}\, D_1 \wedge \bar{x} \,\tilde{\in}\, D_2)$$

$$= \begin{cases} \min\{v(\bar{x} \,\tilde{\in}\, D_1), v(\bar{x} \,\tilde{\in}\, D_2)\} & \text{for} \quad D_1 \cap D_2 \neq \varnothing \\ 0 & \text{for} \quad D_1 \cap D_2 = \varnothing. \end{cases}$$

The function $h(x)$ is called a *certainty distribution*. Let us consider a plant with the input vector $u \in U$ and the output vector $y \in Y$, described by a relation $R(u, y; x) \subset U \times Y$ (*relational knowledge representation*) where the vector of unknown parameters $x \in X$ is assumed to be a value of an uncertain variable described by the certainty distribution $h(x)$ given by an expert. If the relation R is not a function, then the value u determines a set of possible outputs $D_y(u; x) = \{y \in Y : (u, y) \in R(u, y; x)\}$. For the requirement $y \in D_y \subset Y$ given by a user, we can formulate the following **decision problem**: For the given $R(u, y; x)$, $h(x)$ and D_y one should find the decision u^* maximizing the certainty index that the set of possible outputs approximately belongs to D_y (i.e., belongs to D_y for an approximate value of $\bar{x}$). Then

$$u^* = \arg\max_{u \in U} v[D_y(u; \bar{x}) \,\tilde{\subseteq}\, D_y] = \arg\max_{u \in U} \max_{x \in D_x(u)} h(x)$$

where $D_x(u) = \{x \in X : D_y(u; x) \subseteq D_y\}$. It is easy to see that u^* maximizes $v[u \,\tilde{\in}\, D_u(\bar{x})]$ where $D_u(x)$ is a set of all u such that the implication $u \in D_u(x) \rightarrow y \in D_y$ is satisfied. The uncertain variables are dedicated to analysis and decision problems (Bubnicki, 2002, 2004a) in a class of systems containing a decision plant described by a relational knowledge representation with unknown parameter characterized by an expert.

An important example for such a class of decision plants may be a *complex of operations*. It consists of operations characterized by their execution times, and the execution time of a particular operation depends on the amount of a resource allocated to the operation. All the operations use the same kind of a resource which is continuous and may be distributed among operations in any way. In the knowledge-based approach under consideration, this relationship has a form of a relation and an unknown parameter in this relation is assumed to be a value of an uncertain variable characterized by an expert. The decision problem consists then in finding a resource allocation to the operations optimizing a given *performance index* and satisfying the user's requirement typically concerning the execution time of the whole set of operations. Because the resource distribution is based on uncertain knowledge, certainty indexes should be used in decision problem formulations.

Complexes of operations with operations characterized by their execution times are decision plants different than *activity networks* widely used in production or project management (e.g., Banaszak & Jozefowska, 2003). In these networks, the set of activities (production operations or project tasks) is depicted by a graph and the activities are represented by arcs assigned probability distributions describing random durations (execution times) of the activities. The PERT method developed in the 1950s for the analysis of activity networks is based on such a probabilistic description, which seems inadequate in most real world situations. Its extension known as PERT/cost (e.g., Berman, 1964) may be applied also in decision problems, but the allocation is determined in two steps and in the first step typical PERT network analysis and determination of a critical path should be performed. Thus, PERT/cost inherits drawbacks of the PERT method. Models and methods developed for complexes of operations in the 1960s provide analytical formulas and decision algorithms solving resource distribution problems in a unified way on the basis of analytical relationships between operations' execution times and resources allocated to them. If, in the case of an activity network, the execution times are described by experts, the formalism of fuzzy numbers and fuzzy CPM or fuzzy PERT/cost methods may be used (e.g., Mon, Cheng, & Lu, 1995; Fargier, Galvagnon, & Dubois, 2000). If in the case of a complex of operations the parameters in analytical formulas for execution times are described by experts, the formalism of uncertain variables should be used (e.g., Bubnicki, 2003; Orski 2005a, 2005b, 2006a).

In the latter case, for a parallel and for a cascade structure of a complex of operations resource distribution algorithms have been developed (Orski, 2006a), examined (Bubnicki, 2004b; Orski, 2005a), and a method for improving their quality has been proposed (Orski, Sugisaka, & Graczyk, 2006b). The purpose of this chapter is to present a review of problems and algorithms developed in recent years, and to show new results and extensions related to the following directions of current research:

i. Designing knowledge-based resource distribution algorithms for mixed structures of a complex of operations,
ii. Exploiting a concept of three-level uncertainty, which may be considered a knowledge integration method in case of multiple experts, and developing resource distribution algorithms taking into account not only uncertain but also random parameters,
iii. Applying *C*-uncertain variables, and
iv. Applying a learning system to improve the knowledge obtained from the experts.

We present the description of a complex of operations and three formulations of the resource distribution problem. Then, solution algorithms and simple numerical examples are shown for the complex of parallel operations and for the complex of cascade operations. Next, descriptions of typical mixed structures of operations are given, we show how resource distribution algorithms may be derived from formulas determined for parallel and for cascade operations, and present simple numerical examples. We then devote time to the problem of evaluating and improving quality of resource allocation based on experts' knowledge, and include results of simulations. Finally, we address issues related to topics (ii), (iii) and (iv) in the list of current research directions.

KNOWLEDGE REPRESENTATION AND RESOURCE DISTRIBUTION PROBLEMS

Let us consider a complex of k operations described by a set of inequalities

$$T_i \le \varphi_i(u_i, x_i),\ i \in \overline{1,k} \tag{1}$$

where T_i is the execution time of the i-th operation, φ_i is a decreasing function of u_i, u_i is the amount of a resource assigned to the i-th operation, the parameter $x_i \in R^+$ is unknown and assumed to be a value of an *uncertain variable* $\bar{x}_i$ described by a certainty distribution $h_i(x_i)$ given by an expert, and $(\bar{x}_1, ..., \bar{x}_k) \stackrel{\Delta}{=} \bar{x}$ are independent variables.

The total amount of a resource to be distributed among the operations is limited to U, hence every resource allocation $(u_1, ..., u_k) \stackrel{\Delta}{=} u$ must satisfy the constraints:

$$u_i \geq 0 \text{ for each } i \quad \text{and} \quad u_1 + u_2 + ... + u_k = U . \tag{2}$$

Let us denote by T the execution time of the whole complex of operations. It is given by the function

$$T = f(T_1, T_2, ..., T_k) \tag{3}$$

depending on the structure of the complex of operations. In typical situations, functions φ_i in (1) are the following:

$$\varphi_i(u_i, x_i) = \frac{x_i}{u_i}, \quad i \in \overline{1,k} , \tag{4}$$

and certainty distributions h_i are assumed to be triangular, as shown in Figure 1.

For the given values $U > 0$ and $\alpha > 0$ (required project completion time), one may determine the certainty index

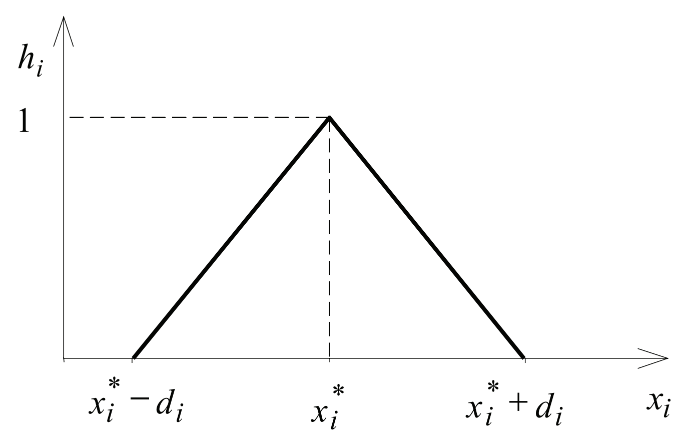
$$v[T(u;\bar{x}) \tilde{\leq} \alpha] \stackrel{\Delta}{=} v(u;\alpha, U) \tag{5}$$

of the property "the execution time T is approximately less or equal to α for the allocation u and the uncertain variable $\bar{x}$" where $T(u;x) = f[\varphi_1(u_1, x_1), \varphi_2(u_2, x_2), ..., \varphi_k(u_k, x_k)]$ is the upper bound function for T. The description of a complex of operations directly corresponds to the description of a decision plant presented in the previous section, that is, allocation u corresponds to plant input, execution time T corresponds to plant output y, the set $(0, \alpha]$ corresponds to the set D_y required by a user, and the inequality $T \leq T(u;x)$ defines the relation $R(u, y; x)$. The following three versions of the **resource distribution problem** may be formulated, depending on which variable in formula (5) is chosen for optimization:

Version I. Given U and α, find u maximizing certainty index v (5), subject to the constraints (2).
Version II. Given U and $\bar{v}$ (*certainty threshold*, that is, $v \geq \bar{v}$ is required), find u minimizing α in (5), subject to the constraints (2).
Version III. Given α and $\bar{v}$, find u minimizing U in (5), subject to the constraints (2).

Performance indexes to be maximized in the three problems stated above are v, α^{-1} and U^{-1}, respectively. Solution algorithms are based on

Figure 1. Triangular certainty distribution for the uncertain variable $\bar{x}_i$



certainty indexes defined individually for particular operations. They are determined (Bubnicki, 2003) by the formulas

$$v_i(u_i;\alpha_i) = v[\varphi_i(u_i,\bar{x}_i) \,\tilde{\le}\, \alpha_i] = \qquad (6)$$

$$\max_{x_i \in D_i(u_i;\alpha_i)} h_i(x_i) = \begin{cases} 1 & \text{for } x_i^* \le \alpha_i u_i \\ \dfrac{1}{d_i}\alpha_i u_i - \dfrac{x_i^*}{d_i} + 1 & \text{for } x_i^* - d_i \le \alpha_i u_i \le x_i^* \\ 0 & \text{otherwise} \end{cases}$$

where

$$D_i(u_i;\alpha_i) = \{x_i \in R^1 : \varphi_i(u_i,x_i) \le \alpha_i\} = (0,\ \alpha_i u_i]\ ,\quad i \in \overline{1,k}\,,$$

and α_i are required completion times for particular operations given directly by a user or needed to be calculated based on α and a structure of the complex of operations.

RESOURCE DISTRIBUTION FOR PARALLEL AND CASCADE OPERATIONS

In this section, main results obtained for a complex of parallel operations and for a complex of cascade operations are shown. The more detailed considerations are presented in Orski (2006). Two typical structures reflecting constraints imposed on a sequence of operations' executions are illustrated in Figure 2. In the case a) operations are executed independently, whereas in the case b) there is a cascade of executions and the execution of any operation may begin only if preceding operations have been completed.

The execution time T of the whole set of operations is given by the following functions (3)

$$T = \max\{T_1, T_2, \ldots, T_k\} \text{ or } T = T_1 + T_2 + \ldots + T_K$$

in the case a) or b), respectively.

Algorithms for Parallel Operations

Version I. Given U and α, find u maximizing v (5), subject to the constraints (2). Because the required execution times α_i are equal to α, $v(u;\alpha,U) = v\{[\varphi_1(u_1,\bar{x}_1) \,\tilde{\le}\, \alpha)] \wedge [\varphi_2(u_2,\bar{x}_2) \,\tilde{\le}\, \alpha)] \wedge \ldots \wedge [\varphi_k(u_k,\bar{x}_k) \,\tilde{\le}\, \alpha)]\} = \min_i v_i(u_i;\alpha)$ and finding the solution

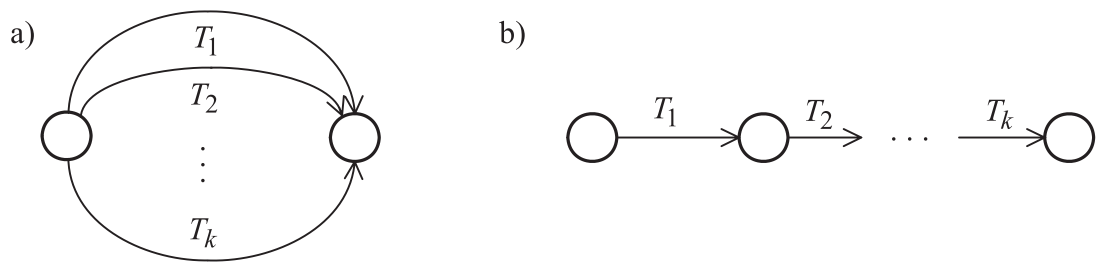
$$u^* = \arg\max_u v(u;\alpha,U)$$

is based on certainty indexes (6). It may be proved (Bubnicki, 2004a) that the optimal distribution should satisfy the set of equations

$$v_1(u_1;\alpha) = v_2(u_2;\alpha) = \ldots = v_k(u_k;\alpha).$$

Hence, it is easy to obtain the following analytical result in the case when $0 < v < 1$:

Figure 2. Operations of a) parallel and b) cascade structure



$$u_i^* = \frac{1}{\alpha}[d_i(\alpha U - \sum_{j=1}^{k} x_j^*)(\sum_{j=1}^{k} d_j)^{-1} + x_i^*] \triangleq \Psi_i(U;\alpha, x^*, d) \quad i \in \overline{1,k} \tag{7}$$

$$v(u^*) = [\alpha U - \sum_{i=1}^{k}(x_i^* - d_i)](\sum_{i=1}^{k} d_i)^{-1} \tag{8}$$

The resource distribution algorithm $\Psi_i(U;\alpha, x^*, d)$ is a linear function of U with parameters depending both on the value α given by a user and on values $(x_1^*, ..., x_k^*) \triangleq x^*$, $(d_1, ..., d_k) \triangleq d$ given by experts.

Version II. Given U and $v = \overline{v}$ (the required *certainty threshold*), find u minimizing α in (5), subject to the constraints (2). Using (8) with $\overline{v}$ instead of $v(u^*)$ one determines the shortest possible execution time

$$\alpha_{\min} = \frac{1}{U}[(\overline{v} - 1)\sum_{i=1}^{k} d_i + \sum_{i=1}^{k} x_i^*] \tag{9}$$

and the optimal allocation

$$u_i^* = \frac{(\overline{v} - 1)d_i + x_i^*}{(\overline{v} - 1)\sum_{j=1}^{k} d_j + \sum_{j=1}^{k} x_j^*} U, \qquad i \in \overline{1,k}. \tag{10}$$

It may be noted that, as in version I, the resource distribution algorithm depends on numerical values given both by a user and by experts.

Version III. Given α and $\overline{v}$, find u minimizing U in (5), subject to the constraints (2). Using (8) with $\overline{v}$ instead of $v(u^*)$ one determines the smallest amount of a resource

$$U_{\min} = \frac{1}{\alpha}[(\overline{v} - 1)\sum_{i=1}^{k} d_i + \sum_{i=1}^{k} x_i^*] \tag{11}$$

and the optimal allocation

$$u_i^* = \frac{(\overline{v} - 1)d_i + x_i^*}{\alpha}, \quad i \in \overline{1,k}. \tag{12}$$

In version III, the resource distribution algorithm depends on numerical values given both by a user and by experts, and is independent of U assumed to be unknown.

Algorithms for Cascade Operations

Version I. One should determine $u^* = \arg\max_u v(u;\alpha, U)$ using certainty indexes (6) and values

$$\alpha_i = \frac{d_i}{u_i}(\alpha - \sum_{j=1}^{k} \frac{x_j^*}{u_j})(\sum_{j=1}^{k} \frac{d_j}{u_j})^{-1} + \frac{x_i^*}{u_i}$$

determined for particular operations by using an optimal decomposition of α (Orski, 2006), that is, such that

$$v_1(u_1, \alpha_1) = v_2(u_2, \alpha_2) = ... = v_k(u_k, \alpha_k) \tag{13}$$

and taking into account that $\alpha_1 + \alpha_2 + ... + \alpha_k = \alpha$. If $0 < v < 1$, it is given by the formula

$$v(u;\alpha, U) = 1 - (\sum_{i=1}^{k} \frac{x_i^*}{u_i} - \alpha)(\sum_{i=1}^{k} \frac{d_i}{u_i})^{-1} \tag{14}$$

which cannot be maximized analytically and a numerical procedure should be used. Based on an analytical solution

$$u_i^* = \frac{\sqrt{x_i^*}}{\sum_{j=1}^{k} \sqrt{x_j^*}} U, \quad i \in \overline{1,k}$$

obtained for a special case of triangular certainty distributions, that is,

$$\frac{x_1^*}{d_1} = \frac{x_2^*}{d_2} = ... = \frac{x_k^*}{d_k},$$

a dedicated numerical procedure for maximization of (14) has been developed and examined (Orski & Hojda, 2007). Numerous computer simulations have shown that it outperforms a Newton method which is known for its fast convergence but requires application of a penalty function for the constraints (2). Our numerical procedure does not need a penalty function and is of much less computational complexity even when compared to a Newton method without a penalty function.

Version II. In a way analogous to that for parallel operations, using (14) with $\bar{v}$ instead of $v(u)$, one determines the shortest possible execution time as the following function of *u*:

$$\alpha_{\min}(u) = \sum_{i=1}^{k} \frac{(\bar{v}-1)d_i + x_i^*}{u_i} \tag{15}$$

Minimization of this function with respect to *u*, subject to the constraints (2), yields

$$\alpha_{\min} = \frac{[\sum_{i=1}^{k} \sqrt{(\bar{v}-1)d_i + x_i^*}\,]^2}{U},$$

$$u_i^* = \frac{\sqrt{(\bar{v}-1)d_i + x_i^*}}{\sum_{j=1}^{k} \sqrt{(\bar{v}-1)d_j + x_j^*}}\, U\,,\ i \in \overline{1,k}\,. \tag{16}$$

Based on the above formulas a numerical-analytical algorithm for maximization of (14) in version I has been developed and examined (Orski & Hojda, 2007). In a numerical part of the algorithm, the first equation in (16) is solved with respect to $\bar{v} = v^*$ for $\alpha_{\min} = \alpha$, and in an analytical part, the optimal allocation is determined by using the second formula in (16) with v^* in the place of $\bar{v}$. Simulation experiments have shown that its performance is comparable to that of the dedicated numerical procedure.

Version III. Using the results (16) obtained in version II, with α instead of $\alpha_{\min}$, one determines the smallest amount of a resource

$$U_{\min} = \frac{1}{\alpha}[\sum_{i=1}^{k} \sqrt{(\bar{v}-1)d_i + x_i^*}\,]^2 \tag{17}$$

and

$$u_i^* = \frac{1}{\alpha}\sqrt{(\bar{v}-1)d_i + x_i^*}\sum_{j=1}^{k} \sqrt{(\bar{v}-1)d_j + x_j^*}\,,\ i \in \overline{1,k} \tag{18}$$

It is worth noting that optimal distribution problems in versions II and III are solvable analytically both for parallel and cascade operation, whereas the problem in version I is more complicated from a computational point of view and a numerical or numerical-analytical procedure should be applied for cascade operations.

RESOURCE DISTRIBUTION FOR MIXED STRUCTURES OF OPERATIONS

In this section, we will show how the formulas and algorithms presented previously may be applied in knowledge-based resource distribution in a complex of operations of neither parallel, nor cascade structure. We will consider simple examples of such a mixed structure and a more general case of a cascade-parallel complex of operations. Generally, for the determination of solution algorithms in mixed structures, for cascade operations one should use formulas presented in the last section, and for parallel ones, the formulas presented in the section preceding it.

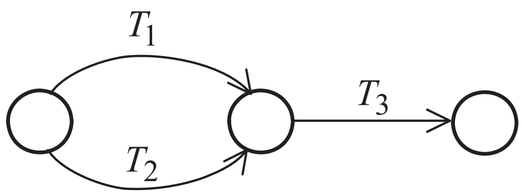
Algorithms for a Simple Mixed Structure ($k = 3$)

Let us take into account a complex of $k = 3$ operations of structure presented in Figure 3. In this case, the formulas corresponding to (1), (2) and (3) are as follows:

$$T_1 \le \frac{x_1}{u_1}, \quad T_2 \le \frac{x_2}{u_2}, \quad T_3 \le \frac{x_3}{u_3},$$

$u_1, u_2, u_3 \ge 0$, $u_1 + u_2 + u_3 = U$ and $T = \max\{T_1, T_2\} + T_3$.

Figure 3. Complex of $k = 3$ operations of a mixed structure



Now, the certainty index (5)

$$v(u;\alpha,U) = v\{[\varphi_1(u_1,\overline{x}_1) \tilde{\leq} \alpha_1] \wedge [\varphi_2(u_2,\overline{x}_2) \tilde{\leq} \alpha_2] \wedge [\varphi_3(u_3,\overline{x}_3) \tilde{\leq} \alpha_3]\} = \min_{i\in\{1,2,3\}} v_i(u_i,\alpha_i)$$

where, according to the structure of the complex in Figure 3, $\alpha_1 = \alpha_2$ and $\alpha_1 + \alpha_3 = \alpha$.

Version I. We may apply (8) for the two parallel operations and introduce $v_{12}(u_1,u_2;\alpha_1) =$

$$1 - \frac{x_1^* + x_2^* - \alpha_1(u_1+u_2)}{d_1+d_2} = 1 - (\frac{x_{12}^*}{u_{12}} - \alpha_1)(\frac{d_{12}}{u_{12}})^{-1}$$

as the aggregated certainty index, where $x_{12}^* = x_1^* + x_2^*$, $d_{12} = d_1 + d_2$ and $u_{12} = u_1 + u_2$. Consequently,

$$v(u;\alpha,U) = \min\{v_{12}(u_{12};\alpha_1),\ v_3(u_3;\ \alpha_3)\}$$

where

$$v_3(u_3;\alpha_3) = 1 - (\frac{x_3^*}{u_3} - \alpha_3)(\frac{d_3}{u_3})^{-1}$$

denotes the certainty index for the one cascade operation obtained from (14). Now, using (14) for the cascade connection of aggregated parallel operations and a single cascade operation one obtains

$$v(u;\alpha,U) =$$

$$= 1 - (\frac{x_{12}^*}{u_{12}} + \frac{x_3^*}{u_3} - \alpha)(\frac{d_{12}}{u_{12}} + \frac{d_3}{u_3})^{-1}$$

$$= 1 - (\frac{x_1^* + x_2^*}{u_1+u_2} + \frac{x_3^*}{u_3} - \alpha)(\frac{d_1+d_2}{u_1+u_2} + \frac{d_3}{u_3})^{-1}$$

Using (2) we substitute $u_{12} = U - u_3$ and maximize $v(u_3;\alpha,U)$ with respect to $u_3 \in (0,U)$. The maximization is reduced to solving a quadratic equation with respect to u_3 and an analytical result may be obtained. However, it is pointless to present it, since we expect that for more complicated mixed structures we will not be able to use it for obtaining analytical solutions, and we will have to use numerical procedures anyway.

Version II. Using (9) for the two parallel operations and using (16) for the one cascade operation yields

$$\alpha_{1,\min} = \frac{(\overline{v}-1)(d_1+d_2) + x_1^* + x_2^*}{u_1+u_2}$$

and

$$\alpha_{3,\min} = \frac{(\overline{v}-1)d_3 + x_3^*}{u_3},$$

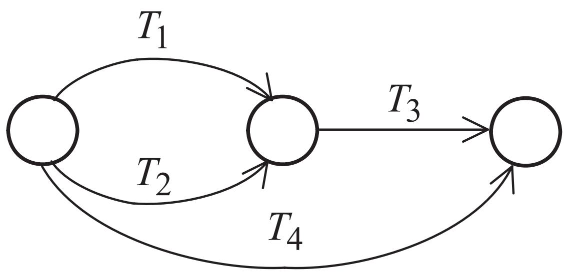
respectively. Again, we may treat the two aggregated parallel operations as a single cascade operation with $x_{12}^* = x_1^* + x_2^*$, $d_{12} = d_1 + d_2$ and $u_{12} = u_1 + u_2$. Let us denote $c_i \stackrel{\Delta}{=} (\overline{v}-1)d_i + x_i^*$, $i \in \{1,2,3\}$ Because $\alpha_{\min} = \alpha_{1,\min} + \alpha_{3,\min}$ then, according to (15),

$$\alpha_{\min}(u) = \frac{c_1+c_2}{u_{12}} + \frac{c_3}{u_3}$$

and using the first formula in (16) yields

$$\alpha_{\min} = \frac{(\sqrt{c_1+c_2} + \sqrt{c_3})^2}{U}$$

Figure 4. Complex of $k = 4$ operations of a mixed structure



From the second formula in (16) we obtain

$$u_3^* = \frac{\sqrt{c_3}}{\sqrt{c_1+c_2}+\sqrt{c_3}}U \text{ , } u_{12} = \frac{\sqrt{c_1+c_2}}{\sqrt{c_1+c_2}+\sqrt{c_3}}U$$

and application of (10) results in

$$u_1^* = \frac{c_1}{c_1+c_2+\sqrt{c_1+c_2}\sqrt{c_3}}U,$$

$$u_2^* = \frac{c_2}{c_1+c_2+\sqrt{c_1+c_2}\sqrt{c_3}}U.$$

Version III. The solution algorithm is based on aggregation analogous to that in version I or in version II. For two aggregated parallel operations from (11) we get $u_{12,\min} = \frac{c_1+c_2}{\alpha_1}$, and for the cascade operation from (17) we have $u_{3,\min} = \frac{c_3}{\alpha_3}$. Then, using formulas (17) and (18) for a complex of cascade operations gives the following results:

$$U_{\min} = \frac{(\sqrt{c_1+c_2}+\sqrt{c_3})^2}{\alpha}, \; u_3^* = \frac{c_3+\sqrt{c_1+c_2}\sqrt{c_3}}{\alpha}$$

$$u_{12,\min} = \frac{c_1+c_2+\sqrt{c_1+c_2}\sqrt{c_3}}{\alpha}$$

Because $u_{12,\min} = \frac{c_1+c_2}{\alpha_1}$, then $\alpha_1 = \frac{\sqrt{c_1+c_2}}{\sqrt{c_1+c_2}+\sqrt{c_3}}\alpha$ and from (12) for two parallel operations we get

$$u_1^* = \frac{c_1(\sqrt{c_1+c_2}+\sqrt{c_3})}{\alpha\sqrt{c_1+c_2}}, \; u_2^* = \frac{c_2(\sqrt{c_1+c_2}+\sqrt{c_3})}{\alpha\sqrt{c_1+c_2}}$$

Example 1. Let $\alpha = 50$, $\bar{v} = 0.8$ and h_1, h_2, h_3 be triangular with $x_1^* = 10$, $d_1 = 5$, $x_2^* = 51$, $d_2 = 10$, $x_3^* = 54$, $d_3 = 20$. Then, in version III one obtains $u_1^* = 0.35$, $u_2^* = 1.89$, $u_3^* = 2.07$ and $U_{\min} = 4.31$.

Algorithms for a Simple Mixed Structure ($k = 4$)

Let us take into account a complex of $k = 4$ operations of structure presented in Figure 4. In this case, the formulas corresponding to (1), (2) and (3) are as follows:

$$T_1 \le \frac{x_1}{u_1}, \quad T_2 \le \frac{x_2}{u_2}, \quad T_3 \le \frac{x_3}{u_3}, \quad T_4 \le \frac{x_4}{u_4}$$

$$u_1, u_2, u_3, u_4 \ge 0, \quad u_1 + u_2 + u_3 + u_4 = U$$

and $\quad T = \max\{\max\{T_1, T_2\} + T_3,\ T_4\}$.

Now, the certainty index (5) $v(u;\alpha,U) = \min_{i\in\{1,2,3,4\}} v_i(u_i,\alpha_i)$ where, according to the structure of the complex in Figure 4, $\alpha_1 = \alpha_2$, $\alpha_1 + \alpha_3 = \alpha$, and $\alpha_4 = \alpha$. Because three of four operations are connected in the same way as in Figure 3, we may use the results obtained for these three operations and apply formulas presented for parallel operations to the fourth operation connected in parallel.

Version I. The certainty index is given by the formula

$$v(u;\alpha,U) = \min\{v_{123}(u_{12},u_3;\alpha),\ v_4(u_4;\ \alpha)\}$$

where $v_{123}(u_{12},u_3;\alpha)$ denotes the certainty index for the complex of three operations, that is,

$$v_{123}(u_{12},u_3;\alpha) = 1 - (\frac{x_{12}^*}{u_{12}} + \frac{x_3^*}{u_3} - \alpha)(\frac{d_{12}}{u_{12}} + \frac{d_3}{u_3})^{-1}$$

and from (8)

$$v_4(u_4;\alpha) = \frac{\alpha u_4 - (x_4^* - d_4)}{d_4} = 1 - \frac{x_4^* - \alpha u_4}{d_4}$$

We cannot use formula (8) further for a parallel connection of the fourth operation and the structure composed of three other operations, because $v_{123}(u_{12}, u_3; \alpha)$ does not represent a certainty index for a single operation being a result of aggregation. We may use (13), which holds also for parallel operations, and maximize $v_{123}(u_{12}, u_3; \alpha)$ subject to the constraints (2) and the additional constraint $v_{123}(u_{12}, u_3; \alpha) = v_4(u_4; \alpha)$. The maximization may be reduced to solving two dependent quadratic equations in such a way that the solution of the first equation is a parameter in the second equation. In general, we will have to use a numerical optimization.

Version II. Using results presented previously for the structure composed of three operations, we have

$$\alpha_{13,\min} = \frac{c_{123}}{u_{123}}, \; \alpha_{4,\min} = \frac{c_4}{u_4}$$

where $u_{123} = u_1 + u_2 + u_3$ is the amount of a resource used in a subcomplex of three operations aggregated in such a way that $c_{123} = (\sqrt{c_1 + c_2} + \sqrt{c_3})^2$ and $\alpha_{4,\min}$ stands for the execution time of the fourth operation parallel to the subcomplex of aggregated operations. Then, using the formula (9) for $\alpha_{\min}$ in case of parallel operations we obtain

$$\alpha_{\min} = \frac{c_{123} + c_4}{U} = \frac{(\sqrt{c_1 + c_2} + \sqrt{c_3})^2 + c_4}{U}$$

and from (10) we obtain

$$u_4^* = \frac{c_4}{(\sqrt{c_1 + c_2} + \sqrt{c_3})^2 + c_4} U,$$

$$u_{123} = \frac{(\sqrt{c_1 + c_2} + \sqrt{c_3})^2}{(\sqrt{c_1 + c_2} + \sqrt{c_3})^2 + c_4} U$$

Now, using previously determined solutions and u_{123} in place of U we get

$$u_3^* = \frac{c_3 + \sqrt{c_1 + c_2}\sqrt{c_3}}{(\sqrt{c_1 + c_2} + \sqrt{c_3})^2 + c_4} U,$$

$$u_{12} = \frac{c_1 + c_2 + \sqrt{c_1 + c_2}\sqrt{c_3}}{(\sqrt{c_1 + c_2} + \sqrt{c_3})^2 + c_4} U$$

$$u_1^* = \frac{c_1(\sqrt{c_1 + c_2} + \sqrt{c_3})}{\sqrt{c_1 + c_2}[(\sqrt{c_1 + c_2} + \sqrt{c_3})^2 + c_4]} U,$$

$$u_2^* = \frac{c_2(\sqrt{c_1 + c_2} + \sqrt{c_3})}{\sqrt{c_1 + c_2}[(\sqrt{c_1 + c_2} + \sqrt{c_3})^2 + c_4]} U.$$

Version III. Using the results for the subcomplex composed of three operations we have

$$u_{123,\min} = \frac{(\sqrt{c_1 + c_2} + \sqrt{c_3})^2}{\alpha}, \; u_{4,\min} = u_4^* = \frac{c_4}{\alpha},$$

and application of (11) as for parallel operations leads to

$$U_{\min} = \frac{(\sqrt{c_1 + c_2} + \sqrt{c_3})^2 + c_4}{\alpha}.$$

The distribution of $u_{123,\min}$ among operations from the subcomplex is, obviously, the same. This results from the fact that now we have one additional parallel operation which needs additional resources to achieve the goal defined in the same way. This would not be the case if additional operation was cascade and not parallel.

Example 2. Let the numerical data be the same as in example 1, and for the additional operation we have a triangular h_4 with $x_4^* = 30$ and $d_4 = 20$. Then, in version III one obtains $u_1^* = 0.35, u_2^* = 1.89$, $u_3^* = 2.07$, $u_4^* = 0.5$ and $U_{\min} = 4.81$.

Algorithms for a Cascade-Parallel Structure

Let us consider a complex of $k = ml$ operations of structure presented in Figure 5. This structure may represent, for example, a supply chain with m production stages. In this multistage production system, a production task at each stage is performed by a cluster (subcomplex) of l production units (operations). Now, we will use double index notation where T_{ij} denotes the execution time of the j-th operation in the i-th subcomplex, u_{ij} is the amount of a resource allocated to this operation, and the unknown parameter x_{ij} is a value of an uncertain variable $\overline{x}_{ij}$ with a given triangular certainty distribution defined by x_{ij}^* and d_{ij}. All uncertain variables are assumed to be independent. The formulas corresponding to (1), (2) and (3) are as follows:

$$T_{ij} \le \frac{x_{ij}}{u_{ij}}, \; i \in \overline{1,m}, \quad j \in \overline{1,l},$$

$$u_{ij} \ge 0, \sum_{i=1}^{m}\sum_{j=1}^{l} u_{ij} = U,$$

and

$$T = \sum_{i=1}^{m} \max_{j \in \overline{1,l}} T_{ij}.$$

The solution of a resource distribution problem in all three versions will now require application of a two-level approach. This approach may be briefly described as finding a solution to the upper level optimization problem (for the whole complex of operations) by solving lower level optimization problems (for particular subcomplexes) and by using these solutions at the upper level. Solving optimization problems at both levels is then coordinated. For particular subcomplexes, formulas presented for parallel operations should be used at the lower level, whereas for the upper level optimization problem we will use formulas presented for cascade operations.

Version I. Using (8) we may express the certainty index for the i-th subcomplex by using U_i and α_i in place of U and α, respectively. Then

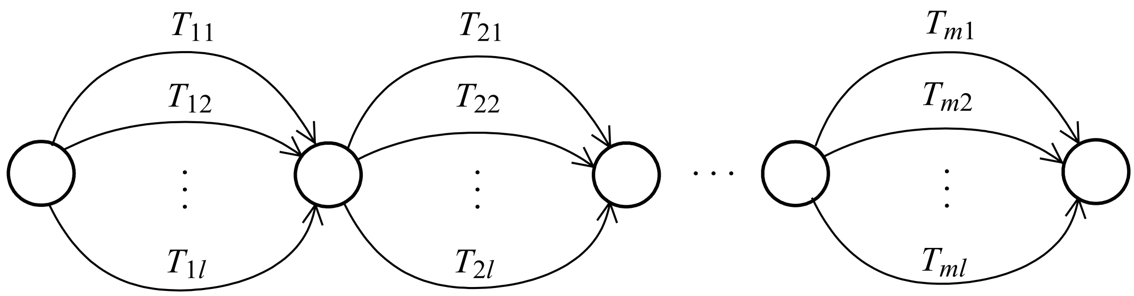
$$v_i(u_{i1}^*, \ldots, u_{il}^*; \alpha_i) = [\alpha_i U_i - \sum_{j=1}^{l}(x_{ij}^* - d_{ij})](\sum_{j=1}^{l} d_{ij})^{-1}$$

where, as in (2), $U_i = u_{i1} + u_{i2} + \ldots + u_{il}$. The parallel operations may be aggregated by introducing $x_i^* = \sum_{j=1}^{l} x_{ij}^*$ and $d_i = \sum_{j=1}^{l} d_{ij}$. Aggregation leads to a definition of the certainty index for the i-th subcomplex the same as for a single operation in (6),

$$v_i(U_i; \alpha_i) = \frac{1}{d_i}\alpha_i U_i - \frac{x_i^*}{d_i} + 1 \cdot$$

A lower level distribution algorithm is analogous to (7), that is,

Figure 5. Complex of $k = m \cdot l$ operations of a cascade-parallel structure



$$u_{ij}^* = \frac{1}{\alpha_i}(d_{ij}\frac{\alpha_i U_i - x_i^*}{d_i} + x_{ij}^*)$$

where α_i is a result of optimal decomposition defined in the same way as for cascade operations with U_i instead of u_i. The amount of a resource in the *i*-th subcomplex U_i is an unknown variable coordinating solution algorithms at the both optimization levels. Its optimal value U_i^* should be determined at the upper level. Because particular subcomplexes are treated as single operations with individual certainty indexes $v_i(U_i;\alpha_i)$, we may apply formula (14) for cascade operations with U_1, U_2, ..., U_m in place of u_1, u_2, ..., u_k which gives

$$v(U_1,...,U_m;\alpha,U) = 1 - (\sum_{i=1}^{m}\frac{x_i^*}{U_i} - \alpha)(\sum_{i=1}^{m}\frac{d_i}{U_i})^{-1}.$$

Maximization of $v(U_1,...,U_m;\alpha,U)$, subject to the constraints $U_i > 0$ and $\sum_{i=1}^{m} U_i = U$, may be performed analytically only under simplifying assumptions, for example, $\frac{x_1^*}{d_1} = \frac{x_2^*}{d_2} = ... = \frac{x_m^*}{d_m} = \gamma$. Then

$$U_i^* = \frac{\sqrt{x_i^*}}{\sum_{p=1}^{m}\sqrt{x_p^*}}U, \; \alpha_i = \alpha\frac{x_i^*}{U_i^*}(\sum_{p=1}^{m}\frac{x_p^*}{U_p^*})^{-1},$$

$$u_{ij}^* = \frac{d_{ij}}{d_i}U_i^* + \frac{x_{ij}^* - \gamma d_{ij}}{\alpha_i}.$$

Otherwise, numerical optimization at the upper level should be performed and its results should be used in analytical resource distribution algorithms at the lower level.

Version II. The solution at the lower level is given by (10) with U_i instead of U, that is,

$$u_{ij}^* = \frac{(\overline{v}-1)d_{ij} + x_{ij}^*}{(\overline{v}-1)d_i + x_i^*}U_i, \qquad i \in \overline{1,m}, \quad j \in \overline{1,l}.$$

Optimal values U_i^* are determined at the upper level, by using the following formula, analogous to the second one in (16):

$$U_i^* = \frac{\sqrt{(\overline{v}-1)d_i + x_i^*}}{\sum_{p=1}^{m}\sqrt{(\overline{v}-1)d_p + x_p^*}}U, \; i \in \overline{1,m}.$$

The shortest execution time is described directly by the first formula in (16).

Version III. The solution at the lower level is given by a formula analogous to (12),

$$u_{ij}^* = \frac{(\overline{v}-1)d_{ij} + x_{ij}^*}{\alpha_i}, \; i \in \overline{1,m}, \quad j \in \overline{1,l},$$

and the smallest amount of a resource in the *i*-th aggregated subcomplex is given by the formula

$$U_{i,\min} = \frac{(\overline{v}-1)d_i + x_i^*}{\alpha_i}.$$

Optimal values of coordinating variables are calculated at the upper level by using (18) for the cascade operations, that is,

$$U_{i,\min}^* = \frac{1}{\alpha}\sqrt{(\overline{v}-1)d_i + x_i^*}\sum_{p=1}^{m}\sqrt{(\overline{v}-1)d_p + x_p^*}, \; i \in \overline{1,m}.$$

The smallest total amount of a resource $U_{\min}$ satisfying a user's requirement is described directly by (17).

Let us, finally, note that the presented simple mixed structure of a complex of $k = 3$ operations may be considered as a special case of the cascade-parallel structure. Therefore, the results presented for that structure in versions I, II and III of a resource distribution problem may be obtained by using formulas presented here, for $m = l = 2$ and under assumption $x_{22}^* = d_{22} = 0$.

KNOWLEDGE QUALITY AND SYSTEM'S PERFORMANCE

If a deterministic model of the complex of operations existed and was known, we might use it for determination of accurate resource distribution algorithms. In the knowledge-based approach presented in this chapter it is, however, assumed that only an uncertain nondeterministic description obtained from human experts is available. Then, it may be expected that quality of a resource allocation determined depends directly on quality of knowledge acquired from the experts. The following sections present a method for evaluation quality of knowledge, show effects of possible inaccuracy in experts' opinions and present a concept of the adaptive resource distribution system in which some parameters may be adjusted so as to improve system's performance.

Quality of Knowledge-Based Resource Distribution

Without a loss of generality, we will discuss and illustrate this important issue for a complex of cascade operations and version I of the resource distribution problem. Let us assume that the exact deterministic descriptions of the operations have a form of the equations

$$T_i = c_i u_i^{-\lambda}, \quad i = 1, 2, ..., k$$

where u_i is the amount of a resource allocated to the i-th operation, $0 < \lambda < 1$. If the parameters c_i are known, the optimal allocation $\overline{u}$ minimizing execution time (3) and satisfying the constraints (2) may be determined in an analytical form

$$\overline{u}_i = \sqrt[\lambda+1]{c_i}\,(\sum_{j=1}^{k} \sqrt[\lambda+1]{c_j}\,)^{-\lambda} U, \quad i = 1, 2, ..., k\ ,$$

and the minimal execution time (3) is given by the formula

$$\overline{T} = (\sum_{i=1}^{k} \sqrt[\lambda+1]{c_i}\,)^{\lambda+1} U^{-\lambda}.$$

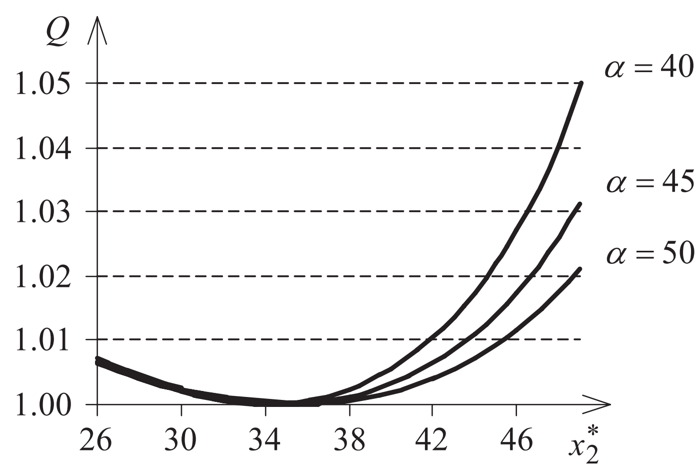
If the values c_i are unknown, we use the description given by experts and apply the allocation u^*. The execution time is then the following

$$T^* = c_1 (u_1^*)^{-\lambda} + c_2 (u_2^*)^{-\lambda} + ... + c_k (u_k^*)^{-\lambda}.$$

For the evaluation of the result of allocations based on experts' knowledge a quality index

$$\frac{T^*}{\overline{T}} \triangleq Q$$

Figure 6. Relationship between Q and x_2^* for different α



may be proposed. The value Q evaluates the allocation based on the knowledge in the form of the inequalities (1) and the certainty distributions h_i (i.e., the knowledge given by experts) and consequently, evaluates the quality of experts. Quality evaluation based on the index Q may be performed for the given values c_i and λ. Hence, it can be used to investigate the influence of the parameters in h_i on the quality of allocation and to compare execution times obtained with different experts. Figure 6 presents the influence of x_2^* on Q for the following data (Orski, 2005b): $U = 1$, $\alpha = 50$, $x_1^* = 4$, $d_1 = 3$, $d_2 = 21$, and for $c_1 = 4$, $c_2 = 36$, $\lambda = 0.95$. It may be observed that for a wide interval of x_2^* the quality of the knowledge-based resource allocation is high enough to be accepted (i.e., $Q \leq 1.02$). However, one can note that within this interval parameters quite strongly influence the quality index, and that there exist optimal values of the parameters, for which $Q \approx 1$. Therefore, further improvement is still possible.

Another observation is that less realistic requirement results in longer execution time. However, unrealistic values of α do not influence quality of allocation based on knowledge of good experts, that is, experts giving near-optimal values of the parameters.

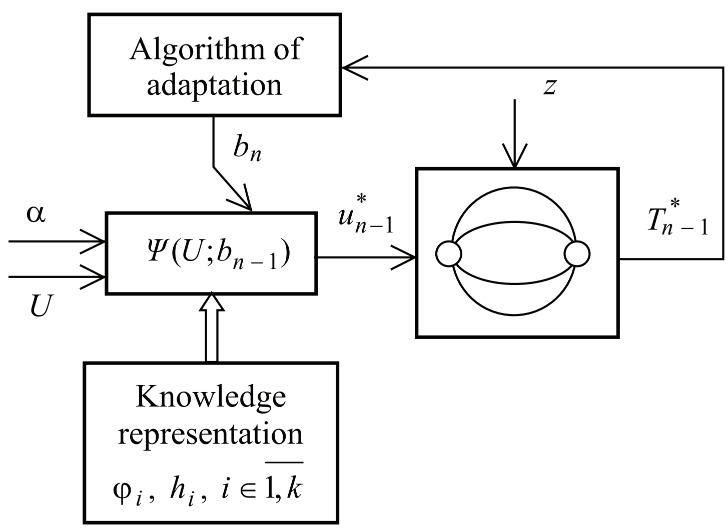
Adaptive Resource Distribution System

Based on expert's knowledge a resource distribution algorithm $\Psi(U;b)$, $b \in B$ is a vector of parameters, should be determined. We have presented resource distribution algorithms determined for different versions of the resource distribution problem formulation and for basic structures of a complex of operations. For example, the resource distribution algorithm $\Psi(U;b)$ determined for the complex of k parallel operations in version I of the problem was given by the following formulas:

$$\Psi_i(U;\alpha, x^*, d) =$$
$$\frac{d_i}{d_1 + \ldots + d_k}\left(U - \frac{x_1^* + \ldots + x_k^*}{\alpha}\right) + \frac{x_i^*}{\alpha}, \; i \in \overline{1,k}\,.$$

Performance of a decision support system using imprecise knowledge is, of course, worse than would be performance of a decision support system using an exact description. Two ways of improving

Figure 7. Adaptive knowledge-based resource distribution system based on uncertain variables



it may be indicated: (i) taking advantage of better experts or (ii) applying adaptation of a knowledge-based resource distribution algorithm Ψ. In case (ii) adaptation requires availability of a real-life complex of operations or its simulator and may consist in *step by step* changing of the parameters *b*, using performance evaluation and starting from the values of *b* resulting from the experts' description. It was suggested that a performance index $Q = T^{*}/\overline{T}$ be used to evaluate performance of a knowledge-based resource distribution algorithm for a complex of parallel operations. Index *Q* may be useful for comparing different experts, under assumption that a deterministic model of the plant is known. In this chapter, we assume that a deterministic model is unknown, so $\overline{T}$ cannot be calculated. However, under assumption that a real-life complex of operations is available, we suggest application of an adaptation process using a performance index $Q = T^{*}$, which leads to an adaptive system, presented in Figure 7.

The following algorithm of adaptation based on stochastic approximation method is proposed

$$b_{n+1} = b_n + \gamma_n \beta_n, \; n = 0,1,...$$

where

$$\beta_{n,i} = \frac{Q(b_n + \overline{\delta}_i) - Q(b_n)}{\delta}, \; i = 1,2,...,r$$

is the estimation of $\frac{\partial Q}{\partial b_i}\Big|_{b_i = b_{n,i}}$; $\overline{\delta}_i$ is a vector of all components equal to 0 except the *i*-th one equal to δ (testing step), and *r* is a number of parameters adjusted in the resource distribution algorithm. The coefficient $\gamma_n > 0$ should satisfy the following relationships to ensure the convergence of an adaptation process

$$\lim_{n\to\infty} \gamma_n = 0, \; \sum_{n=0}^{\infty} \gamma_n = \infty, \; \sum_{n=0}^{\infty} \gamma_n^2 < \infty.$$

The application of the adaptation process is possible if distribution of a resource and execution of the operations are activities performed repeatedly. In the presented above algorithm of adaptation, it is necessary to take into account specific features of the problem under consideration:

1. The algorithm Ψ may be given by three different formulas corresponding to three parts of (6).

Figure 8. Changes of e_n during adaptation for different γ, λ = 1 and α = 8

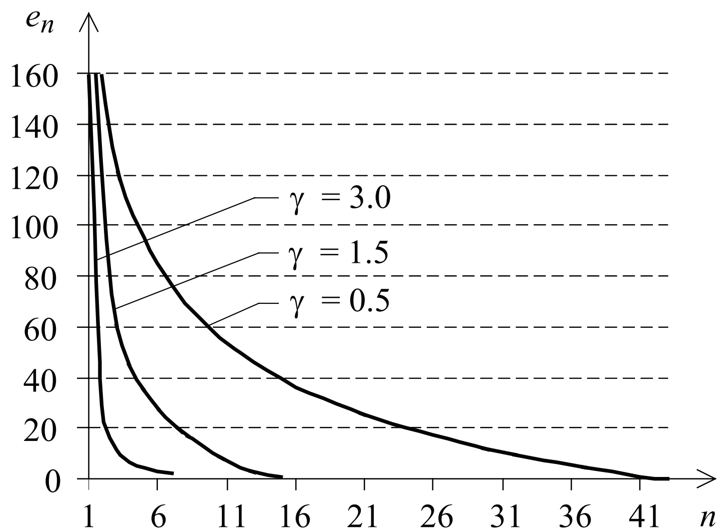
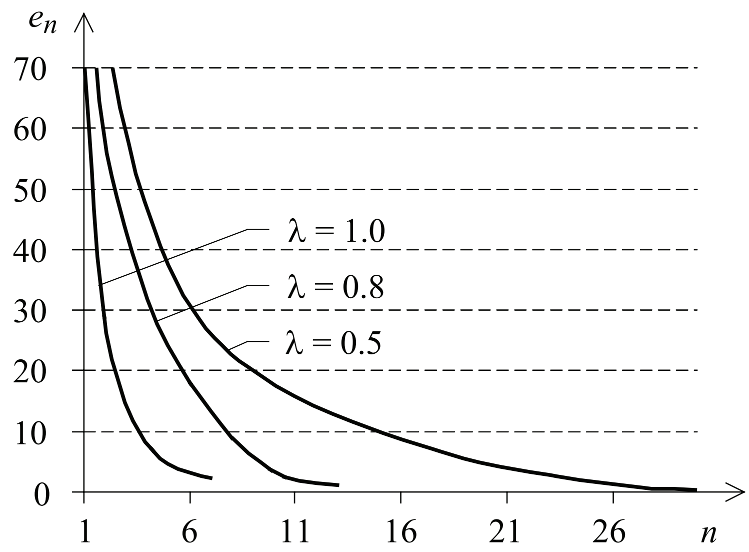


Figure 9. Changes of e_n during adaptation for different λ, γ = 3 and α = 8



2. The constraints (2) should be satisfied in every step of the adaptation process when using a real-life complex of parallel operations.

As components of *b,* we may take $k-1$ of k parameters $\underline{d}_i \triangleq \frac{d_i}{d_1+...+d_k}$, $i \in \overline{1,k}$, and change values of parameters d_i accordingly at each step of the adaptation process, or take $k-1$ of k parameters x_i^*, $i \in \overline{1,k}$, as components of *b*, and change values of parameters d_i when it is necessary to avoid the situation of no solution. In both approaches, changes to d_i cannot cause any changes to values of the parameters $\underline{d}_i$ which are subject to changes introduced by adaptation algorithm only. The second of the suggested approaches has been applied in the example presented below.

Let us consider a complex of two parallel operations of execution times

$$T_1 = \frac{8}{u_1^\lambda}, T_2 = \frac{2}{u_2^\lambda},$$

described by experts in the form of inequalities

$$T_1 \le \frac{x_1}{u_1}, \; T_2 \le \frac{x_2}{u_2}$$

where x_1 and x_2 are values of uncertain variables $\overline{x}_1$ and $\overline{x}_2$ characterized by triangular certainty distributions (Figure 1) with $x_1^* = 8$, d_1=6, $x_2^* = 8$ and $d_2 = 2$. For $U = 1$ (normalized total amount of a resource) a series of simulations of an adaptation process was performed. Different real-life situations were examined by choosing different values of λ. The purpose of the simulations was to investigate the influence of the coefficient γ_n and λ on the convergence of adaptation process and on the quality of a resource distribution algorithm after adaptation.

Figures 8 and 9 show results of simulations obtained for $b = x_1^*$, δ = 0.1 (testing step) and for a value of the adaptation coefficient constant during the adaptation process, that is, $\gamma_n \triangleq \gamma$. The latter assumption makes it impossible to reduce to 0 the error defined as

$$e_n = \frac{T_n^* - \overline{T}}{\overline{T}} \cdot 100\% \;,$$

but is sufficient to satisfy the following stop condition

$$\beta_{n+1} \cdot \beta_n \le 0.$$

According to this condition, the adaptation process is terminated after N steps if the correction to b_N is of a different sign than was the correction to b_{N-1}. The stop condition was suggested based on the observation that with constant γ a satisfactorily small error value was achieved after N adaptation steps during which the adaptation process was monotonic.

We may see that e_n decreases in a monotonic way, which is an advantage of the method proposed. The error decreases faster for larger values of γ and for values of λ being closer to that suggested by an expert.

In the framework of a research on adaptive resource distribution systems a promising approach using artificial neural networks has been developed and reported (Orski et al., 2006). In one method, a specifically designed artificial neural network plays a role of an analytical resource distribution algorithm with weighting parameters adjusted during the learning process, using external trainer (expert) or a real-life complex of operations. In the other method, artificial neural network plays a role of an analytical algorithm of adaptation adjusting parameters of an analytical resource distribution algorithm. In the latter method, a general-purpose multilayer network may be used together with its learning algorithm. Both methods have been verified through a number of simulations which have proven that these methods may be successfully applied also in case where external disturbances influence a real-life complex of operations.

OTHER PROBLEMS AND EXTENSIONS

This section contains a presentation of three concepts which, when desired or necessary, may be applied to a knowledge-based resource distribution. The first one introduces an extension of the knowledge representation by assuming that the parameters in triangular certainty distributions given by experts are values of *random variables*. The second concept consists in employing a *complex definition* of an uncertain variable in place of a *basic definition* of an uncertain variable used to formulate and solve the resource distribution problems. The third concept is similar to that of the adaptive system, but instead of adjusting parameters in the analytical resource distribution algorithm, the parameters of triangular certainty distributions are modified based on an observation of current resource consumption and time elapsed, that is, *learning* a knowledge representation is performed.

Three-Level Uncertainty

Let us consider a set of k operations described by inequalities (1) where T_i is the execution time of the *i*-th operation, u_i is the amount of a resource allocated to the *i*-th operation, an unknown parameter $x_i \in R^1$ is a value of an *uncertain variable* $\overline{x}_i$ described by a certainty distribution $h_i(x_i; w_i)$ given by an expert, and $w_i \in W_i$ is a *random vector variable* $\widetilde{w}_i$ described by probability density $f_i(w_i)$. Both $(\widetilde{w}_1, ..., \widetilde{w}_k) = \widetilde{w}$ and $(\overline{x}_1, ..., \overline{x}_k) = \overline{x}$ are vectors of independent variables. The execution time T is described by a relation defined by (1) and (3) with the vector of uncertain parameters $\overline{x}$ described by $h(x; w)$ where w is a value of a random variable $\widetilde{w}$ described by $f(w)$. Assumption about randomness of w may be justified when we have a representative group of experts randomly chosen from a whole "population" of experts, each of them suggesting values w. Based on their opinions estimates of probability densities $f_i(w_i)$ may be calculated, which results in knowledge integration. We can distinguish two levels of uncertainty concerning the unknown parameters (Bubnicki, 2004a). In fact, we have three levels of uncertainty concerning the complex of operations:

1. Relational level described by (1) and (3).
2. Uncertain level described by $h(x;w)$.
3. Random level described by $f(w)$.

For the given values U and α one may determine the certainty index

$$v[T(u;\bar{x}) \tilde{\le} \alpha] \triangleq v(u;w,\alpha,U)$$

analogous to (5). Then, for the given certainty distributions h_i and probability densities f_i, $i \in \overline{1,k}$, the following three versions of the resource distribution problem may be formulated (Orski, 2007):

Version I. Given U and α, find u maximizing the expected value of $v(u;w,\alpha,U)$, subject to the constraints (2).

Version II. Given U and $v(u;w,\alpha,U)=\bar{v}$ (the required *certainty threshold*), find u minimizing the expected value of α, subject to the constraints (2).

Version III. Given α and $\bar{v}$, find u minimizing the expected value of U, subject to the constraints (2).

Let us present now a solution algorithm to a resource distribution problem formulated in version I for a complex of parallel operations described by (1) and (4), for triangular certainty distributions $h_i(x_i;w_i)$ with random parameters $w_i = (w_{i1}, w_{i2}) = (x_i^*, d_i)$ and $f_i(w_i) = f_{i1}(x_i^*) \cdot f_{i2}(d_i)$, $i \in \overline{1,k}$. One should determine

$$u^* = \arg\max_u \mathrm{E}[v(u;\tilde{w},\alpha,U)],$$

that is, the allocation maximizing an expected value of the certainty index $v(u;w,\alpha,U)$, subject to the constraints (2). Because $v(u;w,\alpha,U) = \min_i v_i(u_i;w_i)$ then $\mathrm{E}[v(u;\tilde{w},\alpha,U)] = \mathrm{E}[\min_i v_i(u_i;\tilde{w}_i)]$, where $v_i(u_i;w_i) = v[\varphi_i(u_i,\bar{x}_i) \tilde{\le} \alpha] = 1-(w_{i1}-\alpha u_i)w_{i2}^{-1}$.

The solution may be based on expected values $\mathrm{E}[v_i(u_i;\tilde{w}_i)] \triangleq e_i(u_i) = a_i u_i + b_i$, where

$$a_i = \alpha \int_{R^1} w_{i2}^{-1} f_{i2}(w_{i2}) dw_{i2},$$

$$b_i = 1 - \int_{R^1}\int_{R^1} w_{i1} w_{i2}^{-1} f_{i1}(w_{i1}) f_{i2}(w_{i2})\, dw_{i1} dw_{i2}$$

may be obtained in an analytical or numerical way. It is easy to note that the optimal allocation should satisfy the set of equations $e_1(u_1) = e_2(u_2) = \dots = e_k(u_k)$. Then, in case $0 < v < 1$:

$$u_i^* = (U + \sum_{j=1}^{k} \frac{b_j - b_i}{a_j})(\sum_{j=1}^{k} \frac{a_i}{a_j})^{-1},\ i \in \overline{1,k},$$

$$e_i(u_i^*) = (U + \sum_{j=1}^{k} \frac{b_j}{a_j})(\sum_{j=1}^{k} \frac{1}{a_j})^{-1}.$$

Application of *C*-Uncertain Variables

Apart from a basic definition of an uncertain variable, which is widely used in all applications of the uncertain variables, a complex definition has been introduced (Bubnicki, 2004a) as the so-called *C*-uncertain variable. According to this definition, the *C*-certainty index of the same property as in (5) would be defined in the following way:

$$v_c[T(u;\bar{x}) \tilde{\le} \alpha] =$$

$$\frac{1}{2}\{ v[T(u;\bar{x}) \tilde{\le} \alpha] + 1 - v[T(u;\bar{x}) \tilde{>} \alpha]\} \triangleq v_c(u;\alpha,U).$$

This means that in the calculation of the certainty index (6) for the *i*-th operation, both

$$D_i(u_i;\alpha_i) = \{x_i \in R^1 : \varphi_i(u_i,x_i) \le \alpha_i\} = (0,\ \alpha_i u_i]$$

and its complement

$$\bar{D}_i(u_i;\alpha_i) = \{x_i \in R^1 : \varphi_i(u_i,x_i) > \alpha_i\} = (\alpha_i u_i, \infty)$$

should be taken into account. Consequently, both parts of $h_i(x_i)$, that is, for $x_i \le x_i^*$ and for $x_i > x_i^*$, will be taken into account, which results in making better use of expert's knowledge. In case of a basic definition of an uncertain variable only one part of the certainty distribution ("half" the expert's knowledge) is used in the determination of the resource allocation. In the determination of $v_c(u;\alpha,U)$ and in the solution of resource distribution problem the following C-certainty indexes for particular operations will be used:

$$v_{ci}(u_i;\alpha_i)=\frac{1}{2}\{v[\varphi_i(u_i,\overline{x}_i)\tilde{\leq}\alpha_i]+1-v[\varphi_i(u_i,\overline{x}_i)\tilde{>}\alpha_i]\}=$$

$$\frac{1}{2}\{\max_{x_i\in D_i(u_i;\alpha_i)}h_i(x_i)+1-\max_{x_i\in\overline{D}_i(u_i;\alpha_i)}h_i(x_i)\}=$$

$$\begin{cases} 1 & \text{for } x_i^*+d_i\leq\alpha_i u_i \\ \frac{1}{2}(\frac{1}{d_i}\alpha_i u_i-\frac{x_i^*}{d_i}+1) & \text{for } x_i^*-d_i\leq\alpha_i u_i\leq x_i^*+d_i \\ 0 & \text{otherwise} \end{cases}$$

A very important and useful theorem may be proved, defining a set of equations which should be satisfied by the solution of a resource distribution problem in version I:

$$v_{c1}(u_1;\alpha_1)=v_{c2}(u_2;\alpha_2)=...=v_{ck}(u_k;\alpha_k).$$

For example, for the complex of parallel operations, applying the above set of equations and definitions of $v_{ci}(u_i;\alpha_i)$ one obtains the same resource distribution algorithm as given by (7) for the basic definition of the uncertain variable. This time, however, (8) is replaced with

$$v_c(u^*)=\frac{1}{2}[\alpha U-\sum_{i=1}^{k}(x_i^*-d_i)](\sum_{i=1}^{k}d_i)^{-1}$$

for $\sum_{i=1}^{k}(x_i^*-d_i)\leq\alpha U\leq\sum_{i=1}^{k}(x_i^*+d_i)$. The difference between applications of basic and C-uncertain variables is more evident in version II of the resource distribution problem. Based on the above formula for $v_c(u^*)$, one obtains

$$\alpha_{c\min}=\frac{1}{U}[(2\overline{v}_c-1)\sum_{i=1}^{k}d_i+\sum_{i=1}^{k}x_i^*]$$

and the optimal allocation

$$u_{ci}^*=\frac{(2\overline{v}_c-1)d_i+x_i^*}{(2\overline{v}_c-1)\sum_{j=1}^{k}d_j+\sum_{j=1}^{k}x_j^*}U,\qquad i\in\overline{1,k}.$$

For example, for $\overline{v}_c=1$ we get $\alpha_{c\min}=\frac{1}{U}(\sum_{i=1}^{k}x_i^*+\sum_{i=1}^{k}d_i)$ and

$$u_{ci}^*=\frac{x_i^*+d_i}{\sum_{j=1}^{k}x_j^*+\sum_{j=1}^{k}d_j}U,$$

whereas for $\overline{v}=1$ the results are as follows: $\alpha_{\min}=\frac{1}{U}\sum_{i=1}^{k}x_i^*$ and

$$u_i^*=\frac{x_i^*}{\sum_{j=1}^{k}x_j^*}U\ .$$

This simple example shows that if maximum certainty threshold is required, then in case of C-uncertain variables both x_i^* and d_i are taken into account, whereas in case of a basic definition of the uncertain variable only the most certain values x_i^* are used.

Learning System for Resource Distribution

The purpose of this ongoing research is to explore possibilities of using actual information on the execution of all operations, obtained at a current moment of time, to update the initial knowledge obtained from experts. The updated knowledge then should be a basis for the determination of the allocation of unused resources, that is, for the redistribution of resources. The problems addressed in the framework of this research are the following:

i. Evaluation of execution of operations completed and of operations being executed;

ii. Knowledge validation;
iii. Knowledge updating by proper adjustment of the certainty distributions, based on the current result of evaluation; and
iv. Resource redistribution based on updated knowledge.

The redistribution of resources referred to in (iv) should be done using resource distribution algorithms presented previously, but issues indicated in (i)-(iii) are new and will be briefly explained below for version III of the resource distribution problem.

(i) Execution Evaluation for a Single Operation

Let us assume that solving the distribution problem (version III) for the whole set of operations with given $\overline{v}$ and α resulted in values u_i^* and α_i, $i \in \overline{1,k}$. Then, *after* execution of the *i*-th operation the following effects may be observed in terms of execution time and resources used:

a. $T_i = \alpha_i$ and $u_i < u_i^*$ (amount of resources smaller than allocated was sufficient); and
b. $T_i \geq \alpha_i$ and $u_i = u_i^*$ (all allocated resources have been consumed).

In version II we would have $T_i \leq \alpha_i$ and $u_i \geq u_i^*$. Both for the case a) and b), based on (6), the performance index

$$\delta_i = \frac{\alpha_i u_i^*}{\overline{v} T_i u_i}$$

may be suggested. It takes a value greater than 1 when the execution time T_i is more optimistic than the required α_i or when the amount of resources actually used u_i is smaller than the amount of resources allocated u_i^*. The above definition accommodates the intuition that the requirement $T_i \leq \alpha_i$ is, in fact, weakened if the certainty threshold $\overline{v} < 1$.

(ii) Knowledge Validation for a Single Operation

On the basis of (1) and (4) one can get the inequality

$$x_i \geq T_i u_i$$

describing possible values of the unknown parameter x_i. This new knowledge resulting from observed values T_i, u_i may be compared to the knowledge given by an expert in the form of $h_i(x_i)$. Let us note (see Figure 1) that only if $T_i u_i \leq x_i^* - d_i$ we may say that the expert's knowledge is valid (thoroughly), and if $T_i u_i \geq x_i^* + d_i$ – that it is invalid (thoroughly), whereas in other cases it may be considered valid to some extent. Then, it seems reasonable to list and analyze all other cases of $T_i u_i$, so as to refer to them when introducing and verifying rules for knowledge updating.

(iii) Knowledge Updating for a Single Operation

We may verify whether the knowledge about x_i used so far is consistent with the current observation and, if not, try to correct it. The similar idea described in Bubnicki (2005) consists in passing to the expert results of current observation and obtaining from her/him a new certainty distribution for the uncertain variable $\overline{x}_i$. In the approach presented here, we assume that the certainty distribution obtained initially from an expert is then corrected automatically, without any further consultation usually unavailable in real-world situations. The learning (knowledge updating) procedure may be based on the performance index δ_i, which is a kind of a measure of discrepancy between the expected effects of the operation and the actual effects observed after its execution. From the economical point of view, we do not want to have $\delta_i > 1$, and would rather use only as much resources as necessary to achieve T_i assumed to be satisfactory to a user. Because $\delta_i < 1$ means that the observed execution time T_i was not satisfactorily short, we may use $\delta_i^* = 1$ as a desired value of the performance index and suggest that a general rule for knowledge updating should be based on the actual error value $\varepsilon_i = (\delta_i - \delta_i^*)$. Then, certainty distribution parameters would be modified so as to reduce ε_i. Particular modification algorithms are under design.

CONCLUSION

The purpose of the research presented in this chapter is to develop methods and algorithms useful for the implementation in a computer system supporting resource distribution in uncertain environments. This computer system based on experts' knowledge, that is, the expert system, could assist managers in an initial phase of planning resource acquisition and distribution on a customer order with possibly imprecisely defined requirements (α, or α and $\bar{v}$). It appears that it would be reasonable and desired to use this expert system in business applications to support managers in the contract negotiation phase.

However, the final step still has to be done. The methodology for determining resource distribution algorithms, presented for simple cases of a mixed structure and for a case of cascade-parallel structure, should be a basis for developing a general resource distribution algorithm solving distribution problems in a uniform way. With this general algorithm, the expert system's designer would not have to go into a detailed analysis of a structure of the complex of operations so as to appropriately perform decomposition or aggregation and then apply algorithms for parallel or cascade subcomplexes.

It is expected that resource distribution problems in versions II and III should be solvable analytically for any mixed structure of a complex of operations, whereas finding resource allocation in version I will always involve numerical or numerical-analytical procedures. Then, the expert system should integrate:

i. Knowledge on the structure of a complex of operations, for example, expressed by using a typical graph theory model like arcs-to-nodes adjacency matrix;
ii. Knowledge on particular operations obtained from experts; and
iii. A general solution algorithm, that is, a knowledge processing algorithm using (i) and (ii).

The general solution algorithm (iii) would further integrate analytical methods for subcomplexes of parallel (versions I, II and III) or cascade (versions II and III) operations, and numerical or numerical-analytical procedures dedicated for the subcomplex of cascade operations (version I).

When implemented, the expert system may be further extended by adding adaptation or learning functionality. Particular methods and algorithms have been already developed and examined by computer simulations (adaptation) or are being developed and will be available in the nearest future (learning). These extensions would allow improvement of system's performance by reducing the initial uncertainty, based on online observation of executions of real-life operations. Therefore, such an extended expert system could assist not only the contract negotiation phase, but also the phase of carrying out the tasks required to fulfill the contract.

Further enhancements of the expert system could be related to the concepts of applying random variables and of applying the formalism of *C*-uncertain variables. These would allow integration of knowledge obtained from multiple experts and better exploration of this knowledge, which could result in better quality of the resource distribution determined and executed even before application of adaptation or learning.

The presented approach may be extended to new and practically important problems, not addressed in this chapter, of (i) resource distribution in complexes of operations with execution times described by two inequalities (instead of a single one) or (ii) resource distribution with a combined performance index, for example, taking into account both execution time and an amount of a resource.

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